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# Decoupling of fields from escaping sources in a de Sitter universe

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**Abstract.** We explore the behaviour of some fields coupled to their respective sources as the latter approach the de Sitter horizon. It is found that the electromagnetic interaction between the source and a typical detector near the origin vanishes as the source is slowly allowed to 'escape' into the horizon. Properties of propagators for a massive vector field, a scalar field and the electromagnetic field are investigated to show how each of these fields in a way decouples from its source resulting in the non-measurability of the associated quantum numbers. This is in contrast to the situation in the Schwarzschild case in which a quasi-static approach of a charge towards the singularity result in a Reisner–Nordstrom black hole. The observer dependence of the decoupling is discussed.

## 1. Introduction

A good deal of progress has been made (Bekenstein 1972, Israel 1967, Price 1972) on the loss of information about the constitution of matter going down a black hole except the total mass, charge and angular momentum—a situation well described by Wheeler as 'a black hole has no hair'. In this article we shall investigate the possibility (and the nature) of the loss of information about (say) a star that approaches the cosmological event horizon in a de Sitter universe. We shall restrict ourselves to the issue of measurability of the number of baryons constituting the source (star) and its total charge.

In the next section we shall deal with the problem of a point charge quasi-statically approaching the de Sitter horizon, following the analogous problem for the Schwarzschild case by Cohen and Wald (1971).

It is found that by requiring the observable invariant of the electromagnetic field to be well behaved at the horizon and at the origin, a unique solution of Maxwell's equations ensues. This resulting solution indicates that as the source approaches the horizon, the electromagnetic field near the origin vanishes. Thus any interaction between the source and a (charged) detector near the origin will vanish. This may be interpreted by the detector as the electromagnetic field itself 'decoupling' from the source.

In § 3 we start by reviewing the general considerations of the decoupling of a typical field from its source as developed by Teitelboim (1972). A quantum field $\Psi$ interacting with a classical source is expressible as

$$\Psi = \Psi^{in} + (\text{coupling constant}) \times I(x) \tag{1.1}$$

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where $I(x)$ is an integral containing the free field retarded propagator, the world line of the source and (sometimes) the field $\Psi$ itself. The behaviour of $I(x)$ as the source–detector distance approaches the de Sitter radius determines whether the resulting field is capable of contributing to a process that may be observable by the detector. In flat space, the propagator is invariantly defined as the Green function of the corresponding wave equation. The Fourier transform in time, moreover, represents outgoing waves for the modes for which wave propagation exists, and exponential damping for the other modes.

An analogous characterisation in the de Sitter space–time can be obtained by resorting to the tortoise coordinate $r^*$ defined by

$$r^* = \frac{a}{2} \ln\left(\frac{a+r}{a-r}\right) \tag{1.2}$$

where $r$ is the usual de Sitter coordinate and $a$ is the radius of the de Sitter horizon.

The transformation (1.2) pushes off the horizon at $r = a$ to $r^* = \infty$ and transforms the metric to

$$ds^2 = -(1 - r^2/a^2)(dt^2 - dr^{*2}) + r^2(d\theta^2 + \sin^2\theta \, d\phi^2). \tag{1.3}$$

In terms of $r^*$, the task of finding a retarded propagator reduces to solving the following radial equation

$$[-\partial^2/\partial t^2 + \partial^2/\partial r^{*2} - V_l(r^*)] \, \Delta(r^*, r^{*\prime}; t - t') = \delta(r^* - r^{*\prime}) \, \delta(t - t') \tag{1.4}$$

where the 'effective potential' $V_l(r^*)$ is strictly positive for finite $r^*$ and goes to a constant value $\mu$ as $r^* \to \infty$, the value depending upon the field under consideration. Further, this potential does not admit any bound states. The retarded propagator is then characterised by the following boundary conditions

$$\Delta(\boldsymbol{x}, \vec{x}'; -\omega) = \Delta^*(\boldsymbol{x}, \vec{x}'; \omega) \tag{1.5a}$$

$$\Delta(\boldsymbol{x}, \boldsymbol{x}'; \omega) \xrightarrow[\substack{r^* \to \infty \\ \omega > 0}]{} \begin{cases} \exp[i(\omega^2 - \mu^2)^{1/2} r^*], & \omega^2 - \mu^2 > 0 \quad (1.5b) \\ \exp[-(\mu^2 - \omega^2)^{1/2} r^*], & \omega^2 - \mu^2 < 0. \quad (1.5c) \end{cases}$$

Equation (1.5$c$) ensures that invariants constructed from the field are bounded at the horizon. Equation (1.4) is solved in § 3 for a massive scalar and a massive vector field in a way that the solutions respect the boundary condition (1.5). It is found that as the source approaches the horizon, $I(x) \to 0$. This implies that the baryon number of the source would not be measurable by a detector near the origin. The observer dependence of the decoupling is mentioned. § 3 shows how some of the results earlier derived for the electromagnetic field can be reiterated by constructing a retarded propagator as a solution to the corresponding equation (1.4). The boundary condition (1.5$c$) is seen to be superfluous as the electromagnetic potential is not an observable quantity and may diverge at the horizon. It is seen how the requirement of boundedness of the observable invariant $\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ can be used to construct the propagator. Unlike the massive meson cases, the 'coupling integral' $I(x)$ need not vanish as the source approaches the horizon to imply 'decoupling' of the electromagnetic field, it suffices $I(x)$ to approach a constant value, as only the derivatives are observable quantities. It is thus that the claims of § 2 are reinforced.

## 2. A point charge in a de Sitter space–time

Consider a point test charge at rest at a point $r = r_0 < a$ along the $z$ axis in a de Sitter space with the metric

$$ds^2 = -(1 - r^2/a^2)\,dt^2 + (1 - r^2/a^2)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2) \tag{2.1}$$

the charge being small enough to render the back reaction on the metric negligible. The Maxwell equation (Misner *et al* 1972)

$$4\pi j^{\mu} = F^{\nu\mu}{}_{;\nu} = (-g)^{-1/2}\frac{\partial}{\partial x^{\nu}}[(-g)^{1/2}F^{\nu\mu}] \tag{2.2}$$

where $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ can easily be seen to reduce to

$$-4\pi j^0 = r^{-2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A_0}{\partial r}\right) + \left(1 - \frac{r^2}{a^2}\right)^{-1}(r^2\sin\theta)^{-1}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_0}{\partial\theta}\right) \tag{2.3}$$

where $j^i$ and $A^i$, $i = r, \theta, \phi$, have been taken to be zero to correspond to the static problem we are considering.

Expanding $A_0$ as

$$A_0(r, \theta) = \sum_{l=0}^{\infty} R_l(r)P_l(\cos\theta)$$

yields for the source free region

$$(1 - r^2/a^2)d/dr(r^2\,dR_l/dr) - l(l+1)R_l(r) = 0. \tag{2.4}$$

The following may be chosen as linearly independent solutions

$$g_l(r) = -\left(\frac{a^2}{r^2} - 1\right)^{1/2} Q_l^{\prime}\left(\frac{a}{r}\right)\frac{(2l+1)!!}{(l+1)!}a^l$$

$$f_l(r) = \begin{cases} r^{-1} & \text{for } l = 0 \\ \dfrac{(l-1)!}{(2l-1)!!}a^{-l-1}\left(\dfrac{a^2}{r^2} - 1\right)^{1/2} P_l^{\prime}\left(\dfrac{a}{r}\right) & \text{for all } l \neq 0. \end{cases} \tag{2.5}$$

where $P_l^{\prime}$ and $Q_l^{\prime}$ are the two Legendre polynomials (Abramowitz and Stegun 1965). The ensuing analysis shall require the following properties of $f_l(r)$ and $g_l(r)$: (a) For $l = 0$, $g_0(r) = 1$ and $f_0(r) = r^{-1}$ (by definition). (b) For all $l$, as $r \ll a$, $g_l \to r^l$ and $f_l \to r^{-(l+1)}$. (c) As $r \to a$, $g_l \to$ finite constant but $dg/dr$ blows up as $\ln(a/r - 1)$ for $l \neq 0$. Also for $l \neq 0$ $f_l(r) \to$ zero as $(a/r - 1)$ but $df_l/dr \to$ finite constant.

To require the total charge of the source to be $e$, the current must have the form (Cohen 1971)

$$j^0 = e\delta(r - r_0)\delta(\cos\theta - 1)$$

For $r < r_0$ and $r > r_0$ equation (2.3) reduces to the source free equation with the general solution

$$A_0(r, \theta) = \begin{cases} \sum_{l=0}^{\infty} (A_l f_l(r) + A_l^{\prime} g_l(r))P_l(\cos\theta), & r > r_0 \\ \sum_{l=0}^{\infty} [B_l^{\prime} f_l(r) + B_l g_l(r))P_l(\cos\theta), & r < r_0 \end{cases} \tag{2.6}$$

Requiring the invariant

$$\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} = (\partial A_0/\partial r)^2 + r^{-2}(1-r^2/a^2)^{-1}(\partial A_0/\partial\theta)^2$$

to be bounded at the origin and at the horizon, we can see, from the behaviour of $f_l$, $g_l$ and their derivatives at the origin and at the horizon, that $A_l' = B_l' = 0$. Further, the continuity of $A_0$ at $r = r_0$ implies

$$A_0(r,\theta) = \begin{cases} \sum_{l=0}^{\infty} c_l g_l(r_0) f_l(r) P_l(\cos\theta), & r > r_0 \quad (2.7a) \\ \sum_{l=0}^{\infty} c_l f_l(r_0) g_l(r) P_l(\cos\theta), & r < r_0. \quad (2.7b) \end{cases}$$

From equation (2.3) and $A_0 = \Sigma\, R_l(r) P_l(\cos\theta)$ we can see from the properties of the Legendre function and after an integration from $r_0 - \epsilon$ to $r_0 + \epsilon$ that

$$-e = c_l/(2l+1) r_0^2 W(g_l, f_l, r_0) \tag{2.8}$$

where $W(g_l, f_l, r_0)$ is the Wronskian of $g_l$ and $f_l$. From the asymptotic form of $f_l$ and $g_l$ at small $r$, it follows that $r_0^2 W(g_l, f_l, r_0) = -(2l+1)$ giving $c_l = e$ thereby completely specifying the solution (2.7).

To look into the implications of this solution, we first note that in the orthonormal frame

$$\omega^0 = (1-r^2/a^2)^{1/2}\,\mathrm{d}t, \qquad \omega' = \mathrm{d}r(1-r^2/a^2)^{-1/2}, \qquad \omega^2 = r\,\mathrm{d}\theta, \qquad \omega^3 = r\sin\theta\,\mathrm{d}\phi,$$

the non-vanishing components of the electromagnetic field tensor $F_{\mu\nu}$ are

$$F_{01} = -F_{10} = -\partial A_0/\partial r; \qquad F_{02} = -F_{20} = r^{-1}(1-r^2/a^2)^{-1/2}\partial A_0/\partial\theta.$$

From these forms, we can determine the interaction of the source with a detector as seen by an observer near the origin. For example for $r > r_0$, $l \neq 0$ and $r \to a$, we get from the property (c) of $f_l$ that $F_{10}$ shall remain finite with $F_{20}$ vanishing as $(a/r-1)^{1/2}$. Thus there shall be a net flux into the horizon. For $r < r_0$ and $l \neq 0$ and with $r$ and $r_0$ both near the horizon, $a$, we can see

$$F_{10}/F_{20} \sim (1-r^2/a^2)^{1/2}\ln(r/a-1) \to 0,$$

i.e. the field shall be mainly tangential. From these considerations and the properties of $f_l$ and $g_l$ we conclude that the field remains well behaved at the horizon as the source approaches it in accordance with our assumption that the charge shall not have a significant back reaction on the metric by drastically affecting the horizon.

As the source approaches the horizon, the field near the origin shall from (2.7$b$) approach zero for the $l \neq 0$ modes and shall be independent of $r$, $\theta$ for the $l = 0$ modes—in both cases giving a null contribution to the observable invariants rendering the charge of the source non-measurable. It may be further noted that although in the frame of the source (say $r'$, $\theta'$, $\phi'$, $t'$) because of the homogeneity and isotropy of the de Sitter spacetime, the original 'detector' previously at $r = 0$ now appears to be at $r' = a$, the field at the horizon from (2.7$a$) does not go to zero, yet an observer stationed at $r' = 0$ cannot determine the charge of the detector as the latter approaches the horizon because of the infinite Doppler shift that will 'screen' the finite motion of the detector due to the finite field at $r' = a$. Thus we conclude that all information about the charge in a star is lost as it crosses the cosmological event horizon in a de Sitter universe.

## 3. Decoupling by propagator analysis

We shall here see how we can study the decoupling of a field from its source by constructing its propagator to satisfy equation (1.5). We start with a scalar meson field.

Consider the scalar meson field interacting with a baryon source of world line $x'(\tau)$ according to the generally covariant interaction

$$(\Box^2-\mu^2-R/6)\phi(x)=\lambda\int_{-\infty}^{\infty}\mathrm{d}\tau\,\delta^4(x-x'(\tau)) \tag{3.1}$$

having a solution

$$\Phi(x)=\phi^{in}(x)+\hat{\phi}(x) \tag{3.2}$$

where

$$\begin{aligned}\hat{\phi}(x)&=\lambda\int_{-\infty}^{\infty}\mathrm{d}\tau\;\Delta_R(x,x'(\tau))\\ &=\lambda(1-r^2/a^2)^{1/2}\,\Delta_R(\boldsymbol{x},\boldsymbol{x}';\omega=0)\end{aligned} \tag{3.3}$$

for a source at rest at the point $x'$; the retarded propagator appearing in (3.3) being a solution of

$$\begin{aligned}(\Box^2-\mu^2-R/6)\;\Delta(x,x')&\equiv[(-g)^{-1/2}\,\partial/\partial x^\alpha((-g)^{1/2}g^{\alpha\beta}\,\partial/\partial x^\beta)-\mu^2-R/6]\;\Delta(x,x')\\ &=\delta^4(x,x')\end{aligned} \tag{3.4}$$

(the results of this section are unaltered by neglecting the $R/6$ term). From the spherical symmetry of the background, the Fourier component of the radial part of the propagator can be seen to satisfy

$$(\partial^2/\partial r^{*2}+\omega^2-V_l(r^*))\;\Delta_R^{(l)}(r^*,r^{*\prime};\omega)=\delta(r^*-r^{*\prime}) \tag{3.5}$$

with

$$V_l(r^*)=(1-r^2/a^2)(l(l+1)/r^2+\mu^2)$$

The radial Green function can be written as

$$\Delta^l(r^*,r^{*\prime};\omega)=W^{-1}\begin{pmatrix}f_2(r^*)f_1(r^{*\prime}), & r^*<r^{*\prime}\\ f_1(r^*)f_2(r^{*\prime}), & r^*>r^{*\prime}\end{pmatrix} \tag{3.6}$$

where $f_1$ and $f_2$ are linearly independent solutions to the equation (3.5) without the $\delta$ function in the right hand side and $W$ is their Wronskian. To satisfy the boundary condition (1.5) we choose

$$\begin{aligned}f_1(r^*)&\xrightarrow{r^*\to\infty}e^{i\omega r^*}\\ f_2(r^*)&\xrightarrow{r^*\to\infty}e^{-i\omega r^*}+R^l(\omega)\,\mathrm{e}^{i\omega r^*}.\end{aligned} \tag{3.7}$$

Using these and the behaviour of the reflection amplitude $R^l(\omega)$ for low frequencies, we can show that for $0\ll r^*<r^{*\prime}$

$$\lim_{r^{*\prime}\to\infty}\Delta^l(r^*,r^{*\prime};\omega=0)=F_0(r^*)$$

where $F_0(r^*)$ is the solution to the zero energy radial equation and goes as $-r^*-a'$ as $r^*\to\infty$ (where $a'$ is a constant depending upon $R^l(\omega)$). Equation (3.3) then implies that the coupling should extinguish as $(1-r'^2/a^2)^{1/2}$ as $r' \to a$. Thus the Yukawa like force between two baryons vanishes as $(1-r'^2/a^2)^{1/2}$ as one of the baryons approaches the horizon, irrespective of where the other baryon is. (Although $F_0(r^*) \to \infty$ as $r^* \to \infty$, the Yukawa force between the two baryons shall still existinguish in the limit as $(1-r^2/a^2)\ln(a-r) \xrightarrow[r\to a]{} 0$).

This result reflects strongly upon the observer dependence of particle quantum numbers such as the baryon number. Two baryons that may be interacting strongly by the exchange of scalar mesons shall, in the frame of an observer who is at the coordinate distance $a$ from any (or both) of the baryons, appear not to be interacting at all. Thus the measurability of the baryon number that depends upon the coupling of the scalar field to the baryon shall become an impossible task by the observer just mentioned in spite of the fact that the baryons may be close to each other and interacting very strongly in any frame sufficiently close to them.

Considering next a vector meson field $\phi^\mu$ coupled to a classical current $j^\mu$ by

$$\phi^{\mu\nu}{}_{;\nu} + \mu^2 \phi^\mu = 4\pi j^\mu \tag{3.8}$$

where

$$\phi_{\mu\nu} \equiv \phi_{(\nu,\mu)}.$$

The solution can be exhibited, as done by Teitelboim (1972), in terms of a bitensor propagator $\Delta^\mu_\nu(x,x')$ that connects the current $j^\mu$ to the field $\hat{\phi}^\mu$ where $\phi^\mu(x) = \phi^{\mu^{(in)}}(x) + \hat{\phi}^\mu(x)$. Considering the source to be static, we shall require only the zero-frequency component of the time transform of the propagator and only the $\Delta_{00}(\boldsymbol{x},\boldsymbol{x}';\omega=0)$ components. In fact for a point source of strength $\lambda$ at rest at $\boldsymbol{x}'$, $\hat{\phi}_0(\boldsymbol{x}) = \lambda\,\Delta_{00}(\boldsymbol{x},\boldsymbol{x}';\omega=0)$. Expanding the propagator as

$$\Delta_{00}(\boldsymbol{x},\boldsymbol{x}';\omega=0) = (rr')^{-1}[(1-r^2/a^2)$$
$$\times (1-r'^2/a^2)]^{1/2} \sum_{l=0}^{\infty} \Delta^l(r^*,r^{*\prime}) \sum_{m=-l}^{l} Y_{lm}(\theta,\phi) Y^*_{lm}(\theta',\phi') \tag{3.9}$$

we get the following equation for $\Delta^l(r^*,r^{*\prime})$:

$$[\partial^2/\partial r^{*2} - V_l(r^*)]\Delta^l(r^*,r^{*\prime}) = \delta(r^*-r^{*\prime}) \tag{3.10}$$

where

$$V_l(r^*) = (1-r^2/a^2)(l(l+1)/r^2+\mu^2) + a^{-2}. \tag{3.11}$$

At the horizon $(r^* \to \infty)$ this potential approaches $a^{-2}$.

Writing as before

$$\Delta^l(r^*,r^{*\prime}) = W^{-1}\begin{cases} f_2(r^*)f_1(r^{*\prime}) & r^* < r^{*\prime} \\ f_1(r^*)f_2(r^{*\prime}) & r^* > r^{*\prime}. \end{cases} \tag{3.12}$$

In order to satisfy the boundary conditions (1.5), we must choose $f_1(r^*) \sim \exp(-r^*/a)$. Thus for $r^* < r^{*\prime}$ we get

$$\Delta^l(r^*,r^{*\prime}) \sim W^{-1}(1-r'^2/a^2)^{1/2} f_2(r^*).$$

This implies that the field decouples faster, namely as $(1-r'^2/a^2)$, as compared to the scalar field case.

For the electromagnetic field, the boundary conditions (1.5), which are constructed in order that the invariants constructed from the fields be bounded at the horizon (except $\hat{\phi}^{\mu}\hat{\phi}_{\mu}$ for the case of the vector field), should be reconsidered, as the electromagnetic potential $A^{\mu}$ is not an observable quantity and need not be bounded at the horizon. Requiring the observable invariant

$$\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} = (\partial A_0/\partial r)^2 + r^{-1}(1 - r^2/a^2)^{-1}(\partial A_0/\partial \gamma)^2,$$

($\gamma$ being the angle between the $(\theta, \phi)$ and $(\theta', \phi')$ directions) to be bounded at the horizon implies that for $l \neq 0$, $f_1(r^*)$ should go as $\exp(-r^*/a)$ as $r^* \to \infty$. This implies that for $r^* < r^{*\prime}$, $\hat{\phi}_0(x) = \lambda f_2(r^*) f_1(r^{*\prime}) \to$ zero as $r \to a$ for all $l \neq 0$.

For $l = 0$, the solution to equation (3.10) (for $\mu = 0$) can be any linear combination of $[(r+a)/(r-a)]^{1/2}$ and $[(r-a)/(r+a)]^{1/2}$ (i.e. $\exp(\pm r^*/a)$) as both these, when combined with equation (3.9), give finite values for the observable invariant at the horizon. Thus the boundary condition at the horizon does not uniquely determine the form of the propagator. If we require the observable invariant to be bounded at the origin as well then it follows that $f_2(r^*)$ must have the form $\exp(r^*/a) - \exp(-r^*/a)$. This implies that

$$\hat{\phi}_0(x) = (4\lambda/rr')[(1 - r^2/a^2)(1 - r'^2/a^2)]^{1/2} \sinh(r^*/a) \cosh(r^{*\prime}/a), \qquad r^* < r^{*\prime}.$$

This approaches a constant value for $r \ll a$ and thereby does not yield any electromagnetic field by which an observer near the origin could determine the charge of the source. We thus obtain the result of § 2 without finding the exact solutions of the Maxwell's equation.

In summary we conclude that the interaction between a source coupling to a scalar or a vector meson field, and a detector, vanishes as the source approaches the horizon with respect to an observer (at the origin). For the electromagnetic field of a point charged source approaching the horizon the electromagnetic potential for the $l \neq 0$ modes vanishes as the source approaches the horizon and attains a constant value for the $l = 0$ mode—in neither case contributing to an observable field.

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